

# Application of the Euler-Bernoulli Beam Equation - Eigenvalue Subproblem

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# The Equation

Our original beam equation:

$$EI \frac{d^4 w}{dx^4} + P^2 \frac{d^2 w}{dx^2} = q,$$

where  $q$  is the sum of the body force and the surface traction per unit length.

In dimensionless form this equation becomes

$$\frac{d^4 w}{dx^4} + B^2 \frac{d^2 w}{dx^2} = 1.$$

We examine the special case:

$$\frac{d^4 w}{dx^4} + B^2 \frac{d^2 w}{dx^2} = 0.$$

# The Equation

For  $q = 0$ , we require both the body force  $q(x)$  and the applied surface traction  $s(x)$  to be 0.

An example of a real-life situation that would produce this equation is provided below:

- If we are considering a beam on the side wall of a mine, the weight and deflection are in perpendicular planes and thus  $q(x) = 0$ .
- If a crack propagates between the beam of interest and its neighbouring beam a gap may form and separate the beams. As a result, there will be no applied surface traction giving  $s(x) = 0$ .

# The Equation

Finding the general solution to our equation yields

$$w(x) = A \cos(Bx) + C \sin(Bx) + \frac{D}{B^2}x + \frac{F}{B^2}$$

subject to the following boundary conditions

$$w(0) = 0,$$

$$w(1) = 0,$$

$$w'(0) = 0,$$

$$w'(1) = 0.$$

# The Equation

Solving our ODE for  $w(x)$  without implementing any restrictions on  $B$  we obtain the trivial solution.

In order to produce non-trivial solutions we proceeded as follows: we first calculate the general solution,

$$w(x) = A \cos(Bx) + C \sin(Bx) + \frac{D}{B^2}x + \frac{F}{B^2}.$$

Then, imposing our boundary conditions we obtained a homogeneous system of equations in the form  $H\underline{x} = \underline{0}$ :

# Solution

$$\begin{pmatrix} 1 & 0 & 0 & B^{-2} \\ \cos(B) & \sin(B) & B^{-2} & B^{-2} \\ 0 & B & B^{-2} & 0 \\ -B \sin(B) & B \cos(B) & B^{-2} & 0 \end{pmatrix} \begin{pmatrix} A \\ C \\ D \\ F \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 0 \\ 0 \end{pmatrix}$$

We want the determinant of the matrix to be equal to 0. In other words,

$$\det(H) = B^5 \sin(B) \left( \tan\left(\frac{B}{2}\right) - \left(\frac{B}{2}\right) \right) = 0.$$

# Solution

Case 1:  $\sin(B) = 0$

Solving for  $B$  gives

$$B = 2n\pi, n \in \mathbb{Z}.$$

We substitute this into the matrix  $H$ :

$$\begin{pmatrix} 1 & 0 & 0 & \frac{1}{(2n\pi)^2} \\ 1 & 0 & \frac{1}{(2n\pi)^2} & \frac{1}{(2n\pi)^2} \\ 0 & 2n\pi & \frac{1}{(2n\pi)^2} & 0 \\ 0 & 2n\pi & \frac{1}{(2n\pi)^2} & 0 \end{pmatrix}$$

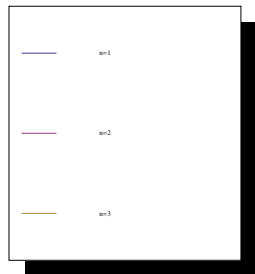
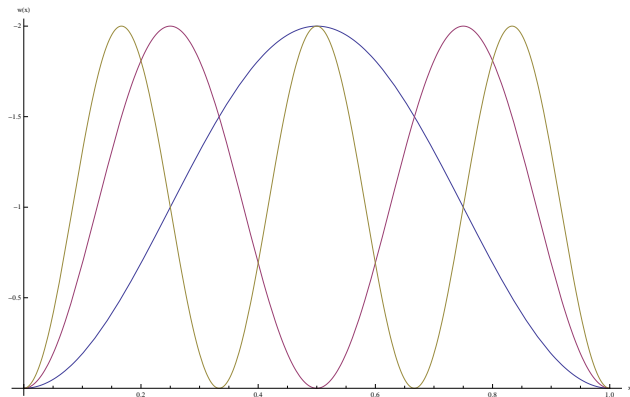
Solving the resulting matrix system with  $B = 2n\pi$  gives

$$w(x) = A(\cos(2n\pi x) - 1),$$

where  $A$  is an arbitrary constant.



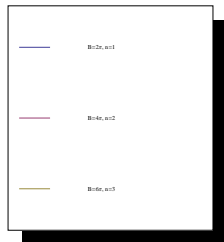
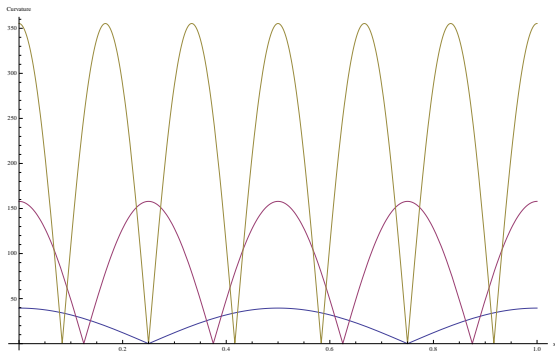
## Plot of the beam deflection



The deflection in the beam is symmetric and all local extrema have the same magnitude.

An observation to be made is that there is a relationship between  $n$  and the number of peaks in the beam's deflection:  $n$  equals the number of peaks present.

## Plot of the beam curvature



The curvature here is also symmetric, and all local extrema have the same magnitude. The extremal values are all the points within the beam that are most likely to experience fracturing.

It can also be noted that the number of extremal values in the curvature increases when the beam number increases.

In conclusion, the following important observations can be made from the results presented

- Possible application to rock bursts
- Predictions of rock burst burst events for specific  $B$  values
- The beam will break in multiple places simultaneously
- The curvature using this method is finite which is possibly more accurate